Mat 2377: Quiz #3

June 13, 2016

Answer all the questions first in one 45 minute sitting. Then check your answers against the solutions given below.

1. An operator receives on average 2 emeregency calls every 3 minutes. Assuming a Poisson process for the number of calls received, what is the probability of receiving 5 calls or more in a 9 minute period?

2. There are 14 multiple choice questions on the midterm. Each question has 5 possible answers only one of which is the correct one. A student decides to guess the answer for each question by choosing one at random. What is the probability of him passing?

3. A roll of a biased die results in a "2" only $\frac{1}{10}$th of the time. What is approximately the probability of having at most seven '2" in 100 rolls of this die?

4. X has a normal distribution with mean 25 and variance 36. Find the constant c such that

$$P\left(|X-25| \leq c\right) = 0.9544$$

5. Let X be the birthweight in gms of babies born in Canada. Assuming that the distribution of X is normal with mean 3315 and variance 575^2 calculate

$$P\left(2584.75 \leq X \leq 4390.25\right)$$

6. If $X\tilde{\ }\mathrm{N}(7,4)$, find $P\left(15.364 \leq (X-7)^2 \leq 20.096\right)$

7. The length of time for one individual to be served at a cafetria is a random variable having an exponential distribution with a mean of 4 minutes. What is the probability that a person will be served in less than 3 minutes on at least 4 of the next 6 days?

8. The lifetime of an electrical component follws an exponential distribution with mean equal to 10 hours. What is the probability that the component will at least 8 hours given it has lasted more than 3 hours?

Solutions

1. We first calculate the mean during a 9 minute period. It is given by

$$\mu = \frac{2}{3} \times 9 = 6$$

From Table A.2 p.434, $P(X \geq 5) = 1 - F(4) = 1 - 0.2851 = 0.7149$

2. This is a binomial type problem. Consider each question as a Bernoulli trial with probability of success equal to $p = \frac{1}{5}$. We want to calculate

$$P(pass) = P(X \geq 7) = 1 - F(6) = 1 - 0.9884 = 0.0116$$

3.This is a binomial probability calculation but the number of repetitions is large so that the tables are not available. Here, $p = 0.10, n = 100$. We use the Poisson approximation with $\mu = np = 10$. Hence, $P(X \leq 7) = 0.2202$

$E[X] = 499\left(\frac{1}{900}\right) - 1\left(\frac{899}{900}\right) = -\frac{4}{9}$

4. We want

$$P\left(\frac{-c}{6} \leq \frac{X - 25}{6} \leq \frac{c}{6}\right) = 0.9544$$

The left hand side is equal to $\Phi\left(\frac{c}{6}\right) - \Phi\left(\frac{-c}{6}\right) = 2\Phi\left(\frac{c}{6}\right) - 1$.Hence,

$$\Phi\left(\frac{c}{6}\right) = \frac{1 + 0.9544}{2} = 0.9772$$

and from table A.3 $\frac{c}{6} = 2$ implying $c = 12$

5. We standardize

$$\begin{aligned} P(2584.75 \leq X \leq 4390.25) &= P\left(\frac{2584.75 - 3315}{575} \leq \frac{X - \mu}{\sigma} \leq \frac{4390.25 - 3315}{575}\right) \\ &= \Phi(1.87) - \Phi(-1.27) \\ &= 0.9693 - 0.1020 = 0.8673 \end{aligned}$$

6. We know that if $X \sim N(\mu, \sigma^2)$, then $\left(\frac{X-\mu}{\sigma}\right)^2 \sim \chi_1^2$. Hence, from Table A.5 p.442

$$\begin{aligned} P\left(15.364 \leq (X-7)^2 \leq 20.096\right) &= P\left(\frac{15.364}{4} \leq \frac{(X-7)^2}{\sigma^2} \leq \frac{20.096}{4}\right) \\ &= P\left(3.841 \leq \chi_1^2 \leq 5.024\right) \\ &= 0.05 - 0.025 = 0.025 \end{aligned}$$

7. This is a two part question. In the first part we calculate the probability that a person will be served in less than 3 minutes. It is

$$p = P(X \leq 3) = \int_0^3 \frac{1}{4} e^{-x/4} dx = 1 - e^{-3/4} = 0.52763$$

In the second part we use the binomial with $n = 6, p = 0.52763$ and compute numerically

$$P(Y \geq 4) = \sum_{x=4}^{6} \binom{6}{x} (0.52763)^x (1 - 0.52763)^{6-x} = 0.39688$$

8. We use the memoryless property for exponential distributions

$$\begin{aligned} P(T \geq 8 | T \geq 3) &= P(T \geq 8 - 3) \\ &= P(T \geq 5) \\ &= \int_5^\infty \frac{1}{10} e^{-x/10} dx \\ &= e^{-1/2} = 0.60653 \end{aligned}$$